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STRUCTURAL DAMPING IN SUSPENSION BRIDGES

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PAPERS

STRUCTURAL DAMPING IN SUSPENSION BRIDGES

By the Late Friedrich Bleich,[1] M. ASCE, and L. W. Teller,[2] M. ASCE

Synopsis

The collapse of the Tacoma Narrows Bridge in Washington in 1940 focused attention on the importance of a better knowledge of the capacity of suspension bridges to dissipate energy imparted by wind action. This paper reports part of the investigational work undertaken by the Advisory Board on the Investigation of Suspension Bridges. It comprises: (1) A theoretical study of damping capacity of suspension bridges resulting from internal friction and from various sources of dry or Coulomb friction in the structure; (2) an account of an extensive laboratory study of frictional damping in structural members; and (3) a correlation of the theory with experimental data. The paper suggests that a substantial amount of additional damping can be provided without resort to special damping devices if consideration is given in design to the potential sources of friction available in the floor system.

PART I. THEORETICAL STUDY OF THE DAMPING CAPACITY OF SUSPENSION BRIDGES

Experiments on the behavior of suspension bridges under the action of wind, which have been conducted for several years, disclose that the capacity of a vibrating bridge structure to dissipate the energy fed into it by pulsating wind forces, in general, has an important bearing on the dynamic stability of the structure.

It is questionable whether information on structural damping in suspension bridges could be obtained by field tests on existing long-span bridges because of

Note.—Written comments are invited for publication; the last discussion should be submitted by September, 1951.

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the prohibitive costs, even if suitable methods could be devised for vibrating the formidable mass of such a bridge. Moreover, such field tests can only yield data on the over-all damping capacity of the structure, a capacity which may vary to a large extent from bridge to bridge; they cannot provide the basic information on the actual sources of the energy-dissipating capacity of the structure, or on the real nature of the damping phenomena in suspension bridges. Nevertheless, such detailed information is necessary in studying ways and means to insure the required damping capacity in any given bridge design.

It must be realized also that laboratory tests on full models cannot furnish real information on the damping capacity of the prototype because it is impossible, as a practical matter, to duplicate the structure of a bridge to a reduced scale in such detail as to simulate, reliably, the condition of friction existing in the prototype.

However, a theoretical approach is possible for solving the rather simple problem of damping in a bridge structure if basic experimental data are available as to the logarithmic decrement of internal damping in rolled, riveted, and welded steel structures, and as to the amount of friction that can be safely assumed as acting between the sliding parts of the structure. The following theoretical study, in its essentials, is a condensation of a report submitted to the Advisory Board on the Investigation of Suspension Bridges (see "Acknowledgment").

Damping in General

"Damping" is the dissipation of the energy that is imparted to a vibrating structure by exciting forces, whereby one part of the external energy is transformed into molecular energy and another part is dissipated to surrounding objects or atmosphere. When damping can be localized at certain points or regions of the structure, damping may be thought of as being caused by forces acting in those points or regions and opposing the motion. These forces are referred to as damping forces.

"Damping capacity" of a structure may be defined as the ratio of the energy ΔW dissipated on one cycle of oscillation to the maximum amount of energy W accumulated in the structure in that cycle. Therefore, the damping capacity ψ, being a nondimensional quantity, is expressed as

$$\psi = \frac{\Delta W}{W} \qquad (1)$$

Suspension bridges are built up of several structural elements, such as cables, stiffening trusses, and towers. Designating the energy losses per cycle of the various elements by ΔW_1, ΔW_2, $\cdots$, respectively, the damping capacity of the entire system, according to the foregoing definition, is determined by

$$\psi = \frac{\Delta W_1 + \Delta W_2 + \cdots}{W} \qquad (2a)$$

in which W is the energy of oscillation of the entire system. Eq. 2*a* remains unchanged in writing

$$\psi = \frac{\Delta W_1}{W_1}\frac{W_1}{W} + \frac{\Delta W_2}{W_2}\frac{W_2}{W} + \cdots \qquad (2b)$$

in which W_1, W_2, $\cdots$ denote the energies of oscillation of the respective elements. Since $\psi_1 = \frac{W\Delta_1}{W_1}$, $\psi_2 = \frac{\Delta W_2}{W_2}$, etc., represent the individual damping capacities of the elements and $r_1 = \frac{W_1}{W}$, $r_2 = \frac{W_2}{W}$, etc., the relative energy storage capacities of these elements, the over-all damping capacity ψ of the structure, is expressed by

$$\psi = r_1 \psi_1 + r_2 \psi_2 + \cdots \qquad (3)$$

This linear relationship is based on the tacit assumption that ψ_1, ψ_2, $\cdots$ are small as compared with unity so that the frequency of the damped vibration of the system can be considered as being identical with the frequency of the undamped oscillation. Eq. 3 is the fundamental relation for the computation of the damping capacity of a compound structure from the damping capacities of its elements.

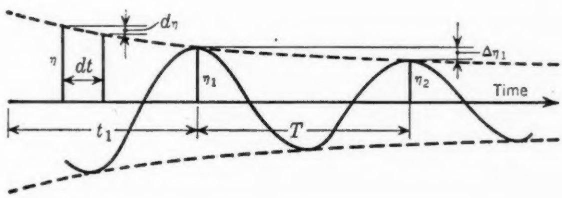


FIG. 1

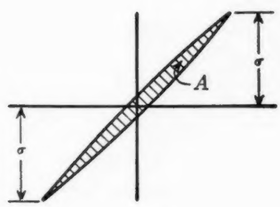


FIG. 2

Consider the amplitude-time diagram of a damped periodic motion as shown in Fig. 1. Let W_1 be the potential energy at the instant t_1 and W_2, the energy at the instant $t_1 + T$, in which T is the period of the oscillation. Assuming small motion and the validity of Hooke's law, the maximum amount of the potential energy of the structure in motion is proportional to the square of the amplitude η, so that $W = k\,\eta^2$. The difference between the potential energy W_1 at the time t_1 and the energy W_2 at the time $t_1 + T$, one cycle later, gives the loss of energy (dissipated energy) or the total damping of the structure for one cycle. Hence the damping capacity ψ may be expressed by $\psi = \frac{W_1 - W_2}{W_1}$ $= \frac{k\,\eta^2_1 - k\,\eta^2_2}{k\,\eta^2_1} = 1 - \frac{\eta^2_2}{\eta^2_1}$; or

$$\psi = 1 - \frac{(\eta_1 - \Delta\eta_1)^2}{\eta^2_1} = 2\,\frac{\Delta\eta_1}{\eta_1} \qquad (4)$$

—neglecting $\frac{\Delta\eta^2_1}{\eta^2_1}$ as a small quantity of the second order. To obtain the equation of the curve, indicated by dotted lines in Fig. 1 (which determines the decay of the amplitudes), consider an infinitely small part of the curve between t and $t + dt$. Assuming ψ to be constant and applying Eq. 4 to the infinitely

small interval dt, the differential equation is

$$-\frac{d\eta}{\eta} = \frac{\psi}{2}\frac{dt}{T} \quad (5a)$$

Integration furnishes

$$\log_e \eta = -\frac{\psi}{2}\frac{t}{T} + C \quad (5b)$$

from which

$$\eta = e^{C-\psi t/(2T)} = \eta_0 e^{-\psi t/(2T)} \quad (6)$$

is derived readily. In Eq. 6, η_0 is the amplitude at the start of vibration at the time $t = 0$. Considering two consecutive maxima η_r and η_{r+1} of the amplitude-time curve, having the distance $t = T$ and applying Eq. 6, the significant relation—

$$\frac{\eta_{r+1}}{\eta_r} = \frac{e^{-\psi(t_r+T)/(2T)}}{e^{-\psi t_r/(2T)}} = e^{-\psi/2} \quad (7)$$

—is obtained. Eq. 7 expresses the rate of decay of the amplitudes. The ratio between consecutive maxima is constant; the amplitudes decrease in a geometric series. The quantity $\delta = \psi/2$ is dimensionless and is known as the logarithmic decrement of damping because

$$\log \frac{\eta_{r+1}}{\eta_r} = -\delta \quad (8)$$

The relation $\delta = \psi/2$ is to be considered an approximation and applies only when δ is small compared with unity. In deriving the relation $\delta = \psi/2$, the damping capacity ψ was assumed to be a constant. In general, in steel structures ψ varies with the decaying amplitude of the vibration, and the actual curve of decay departs markedly from an exponential curve. Nevertheless, the relation $\delta = \psi/2$ also holds in cases of varying ψ since this magnitude, without appreciable error, can be considered invariable during one cycle.

In suspension bridges the damping has its origin chiefly in the following sources:

1. The imperfect elesticity of the material of the structure—that is, elastic hysteresis of the structural material. (In suspension bridges of the conventional type vibrations change the stresses in the cables only insignificantly; damping from this source is negligible, therefore, and has not been considered in this paper.)
2. Plastic yielding and friction due to small relative displacements in the riveted joints of the structure.
3. Internal friction in the concrete floor slab of the bridges.
4. Friction in the main expansion joints of the floor structure and in the secondary joints, or at the pedestals (where the stringers and other parts slide on their supports at the floor beams); also, frictional resistance at the cable saddles and at suspender connections.
5. Friction at the end bearings of the trusses or in the installed friction devices.
6. Resistance of the air.

The damping effect caused by the damping influences, sources 1, 2, and 3, is referred to as "internal damping." Sources 4 and 5 represent typical "Coulomb damping" or "dry friction" and, finally, resistance of the air causes "atmospheric damping." In this paper structural damping only, generating from sources 1 to 5, will be discussed.

Internal Damping

"Internal damping" is the dissipation of energy as a result of friction between the molecules of the material or between the faying surfaces of structural parts riveted or bolted together. The stress-strain curve for a specimen subjected to a cycle of loading, within the range of the proportional limit, is not coincident with that for unloading, and neither curve is an exact straight line, as Hooke's law postulates. In fact, a hysteresis loop occurs (Fig. 2), and the area A of the loop indicates the amount of energy dissipated during a completely reversed cycle. The departure from Hooke's law increases with increasing limiting stress σ, which means that the ratio of the amount of dissipated energy to the total elastic energy supplied to the specimen increases when the limiting stress σ involved in the vibration rises.

The energy dissipated by the elastic hysteresis of the steel is obviously small because of the slight departure of the stress-strain curve from a straight line, at least within the range of the small dynamic stresses which may occur in a suspension bridge and which do not exceed a few thousand pounds per square inch. The logarithmic decrement δ of steel specimens as a result of the elastic hysteresis of the material was found to be very small. For dynamic stresses between 1,000 lb per sq in.[2] and 3,000 lb per in.[2], δ varies only slightly and may be assumed to have an average value of 0.004.

The dissipation of energy is markedly increased in the riveted members of bridge structures because of the plastic slip in the riveted connections. Detailed information on the magnitude, and variation with stress, of the logarithmic decrement as observed on a model truss will be presented in Part II. The various tests showed that, in so far as small dynamic stresses between 1,000 lb per sq in. and 3,000 lb per sq in. are concerned, the logarithmic decrement δ may reach an average value about twice or three times the value of δ as observed on rolled sections.

Since elastic hysteresis in a wide range is nearly independent of the velocity of the oscillations, the internal damping capacity ψ can be considered as being independent of the frequency of the vibrating structures.

Coulomb Damping

It may be concluded from the previous section on "hysteresis damping" that the internal damping capacity of steel bridges is comparatively small. Observations on existing steel bridges of various types indicate damping capacities of substantially higher order than those related to internal damping. The conclusion must be drawn that a considerable part of the damping effect in a bridge structure is due to energy dissipation by internal or external dry friction.

Relatively simple theoretical studies actually reveal that Coulomb friction may play an important role in the over-all damping capacity of suspension

bridges. The fact must be stressed that the effect of Coulomb friction on the damping capacity can be substantiated fairly well by theoretical considerations. The theory can be based on the experimental data as to the amount of the coefficient of dry friction, data which may be obtained in each individual case by relatively simple tests as described in Part II.

The effect of Coulomb friction on an oscillating structure is different from that of internal damping. Whereas the internal damping capacity slowly increases with rising amplitude, the frictional damping capacity (as will be shown subsequently) remains constant or decreases when the amplitude increases. In most cases, Coulomb damping reaches its maximum value at the beginning of a forced or self-excited vibration. This is a very significant fact which makes Coulomb friction an important factor in the over-all damping capacity of a bridge structure, particularly in the light of the fact that the rate of dissipation of energy by Coulomb damping often exceeds substantially the rate of energy dissipation by internal damping.

Coulomb Damping in an Oscillating Simple Span

It is convenient to discuss the theory of frictional damping in a simple span to obtain the basic concept for the analysis in the case of suspension bridges. Application of the developed method of approach to suspension bridges will prove to be a matter of somewhat more analysis. Two fundamental problems will be investigated hereinafter: (1) Damping capacity caused by friction in the end bearings and (2) damping capacity due to friction between the stringers and the floor beams—the stringers being assumed to slide on the top of the floor beams.

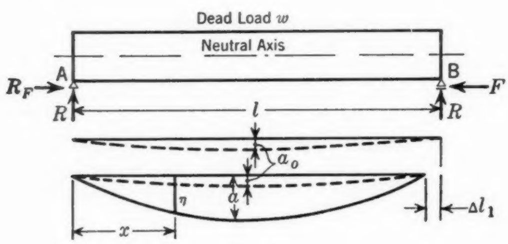


Fig. 3

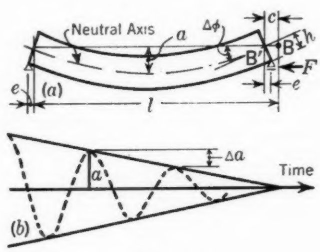


Fig. 4

Damping Caused by Friction in the End Bearings.—Consider the span shown in Fig. 3, acted on by the dead load w and frictional force F on the sliding bearing at point B. Force F is given by the product $R f$, in which R is the vertical reaction at point B due to dead load w and the varying inertia forces of the vibrating bridge. The coefficient of friction f is assumed to be a known constant. It is assumed that the bridge oscillates in the first symmetric mode as indicated in Fig. 3. At the start of the forced or excited vibration, when the amplitude is small, the trusses behave like two-hinged arch ribs producing a horizontal thrust H. In the first phase, H increases with increasing amplitude until it reaches the value of F at the amplitude a_o, at which the frictional resistance is overcome. When a becomes greater than a_o horizontal motion

starts at point B until the truss reaches its extreme amplitude a. The path Δl_1 of point B, being a function of a, determines the energy $\Delta W_1 = F\,\Delta l_1$ dissipated in the first quarter of the cycle of vibration.

Force F varies during one cycle because the reaction R is the algebraic sum of R_w (reaction due to w) and R_I (reaction due to the inertia forces). It is easily seen, however, that R increases, in one half the cycle, to $R_w + R_I$ and decreases in the other half to $R_w - R_I$. The average value of R is equal to R_w. Hence, F may be considered a constant, $F = R_w f$.

The symbol ΔW is defined as the total energy dissipated in a complete cycle of vibration. The total path Δl_1 of the end point B of the truss during the first quarter of the cycle consists of two parts. The first part (referring to Fig. 4(a)) is the displacement $\overline{BB'} = c$ of the end cross section of the bridge as a result of the curvature of the neutral axis. The second part is caused by the elongation $2e$ of the bottom chord. The total path Δl_1, therefore, is given by $\Delta l_1 = 2\,e - c$, assuming $2\,e > c$. When the girder swings back into its original position point B moves in the opposite direction. Again the displacement is $\Delta l_2 = 2\,e - c$, the small diminution of the amplitude caused by the damping being neglected. Similar reasoning shows that the path Δl during the third quarter and the fourth quarter of the cycle is $\Delta l_3 = \Delta l_4 = 2\,e + c$. Thus, the work done by the force F during one complete cycle is determined by

$$\Delta W = 2\,F\,(2\,e - c + 2\,e + c) = 8\,F\,e \quad \text{(9a)}$$

from which it can be assumed that F already has its full value at the beginning of each quarter of the cycle.

In the foregoing section it was assumed that c is smaller than $2\,e$ if $c > 2\,e$ (and it can be), the displacement Δl in the first and second quarters of the cycle is defined by $c - 2\,e$, and by $c + 2\,e$ in each of the two other quarters of the cycle; ΔW is now controlled by c and becomes

$$\Delta W = 2\,F\,(c - 2\,e + c + 2\,e) = 4\,F\,c \quad \text{(9b)}$$

The subsequent paragraphs will show that Eqs. 9 correspond to two different types of damping, referred to as the first and second types of Coulomb damping.

First Type of Coulomb Damping.—This type controls (for instance) the symmetric modes of a vibrating beam. The displacement $2\,e$ is given by

$$2\,e = 2\,h\,\Delta\phi \quad \text{(10)}$$

in which h is the distance of the neutral axis from the sliding surface of the bearing; and $\Delta\phi$ is the slope of the center line at cross section B (see Fig. 4(a)). The displacement form of the first symmetric mode of a simple beam is given by

$$\eta = a \sin \frac{\pi\,x}{l} \quad \text{(11)}$$

Therefore,

$$\Delta\phi = \frac{\pi}{l}\,a \quad \text{(12a)}$$

and

$$e = \pi\,\frac{h}{l}\,a \quad \text{(12b)}$$

Substituting Eq. 12*b* in Eq. 9*a*,

$$\Delta W = 8\,\pi \frac{h}{l} F\,a \qquad (13)$$

The maximum value of the total potential energy stored in the vibrating beam in its extreme position is

$$W = \frac{E\,I}{2} \int_o^l (\eta'')^2\,dx = \frac{\pi^4\,E\,I}{4\,l^3}\,a^2 \qquad (14)$$

in which I is the moment of inertia of the beam. Hence, the damping capacity ψ is determined by

$$\psi = \frac{\Delta W}{W} = \frac{32\,h\,l^2\,F}{\pi^3\,E\,I\,a} \qquad (15)$$

Eq. 15 indicates that ψ decreases when the amplitude a of the vibration increases. The damping capacity reaches its maximum value at the instant when a becomes greater than a_o; that is, at the limiting amplitude, when friction in the bearing is overcome. In other words, Coulomb friction of the first type has its maximum damping effect at the starting phase of a forced vibration.

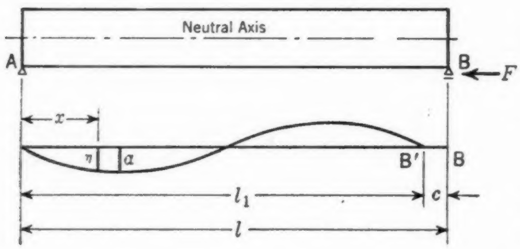


FIG. 5

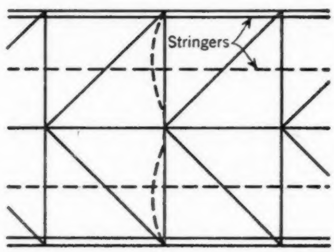


FIG. 6

Since Eq. 4 has general validity,

$$\psi = \frac{\Delta W}{W} = 2\,\frac{\Delta a}{a} \qquad (16)$$

in which Δa is the difference of the amplitudes of two consecutive cycles. Equating Eqs. 15 and 16,

$$\Delta a = \frac{16\,h\,l^2}{\pi^3\,E\,I} F \qquad (17)$$

Eq. 17 shows that Δa is constant, which means that the diagram of decay of the amplitude is a straight line, as shown in Fig. 4(*b*).

Second Type of Coulomb Damping.—The asymmetric modes of vibration are controlled by the second type of damping. Consider a beam AB (Fig. 5) oscillating in the first asymmetric mode. The elongation $2\,e$ of the bottom chord is zero and the work done by the friction force F depends only on the

displacement c caused by the curvature of the center line. Assuming $\eta = a \sin \frac{2\pi x}{l}$, the difference c between the span l and the length l_1 of chord AB′ is

$$c = l - l_1 = \pi^2 \frac{a^2}{l} \qquad (18)$$

the energy dissipated in the sliding bearing, in one cycle of vibration, therefore, is

$$\Delta W = 4 F c = 4 \pi^2 \frac{F}{l} a^2 \qquad (19a)$$

The potential energy accumulated in the beam amounts to

$$W = \frac{4 \pi^4 E I}{l^3} a^2 \qquad (19b)$$

Combining Eqs. 19*a* and 19*b* furnishes the damping capacity—namely,

$$\psi = \frac{\Delta W}{W} = \frac{4 \pi^2 F a^2}{l} \frac{l^3}{4 \pi^4 E I a^2} = \frac{l^2 F}{\pi^2 E I} \qquad (20)$$

In Eq. 20, ψ is independent of the amplitude and may be considered a constant for any given design and mode.

Internal damping of small vibrations is characterized by a constant logarithmic decrement. The amplitude-decay diagram, practically, is an exponential curve, a type of damping frequently referred to as viscous damping. Therefore, friction damping of the second type, as described by Eq. 20, is the viscous type of damping, and the logarithmic decrement determining the decay of the amplitude is given by

$$\delta = \frac{1}{2} \frac{\Delta W}{W} = \frac{l^2 F}{2 \pi^2 E I} \qquad (21)$$

The value of ψ as given by Eq. 20 is small compared with the value of ψ according to Eq. 15. The effect of Coulomb damping due to friction at the end bearings, in case of asymmetric vibration, may be considered negligible.

Damping Due to Friction Between Stringers and Floor Beams

Assume that the stringers of a long single-span bridge are sliding on stringer pedestals on top of the floor beams. When the bridge deflects, a relative displacement of stringer and floor beams takes place as a consequence of the difference in length between the undistorted stringers and the distorted top chords of the trusses. In calculating the total amount of the energy dissipated by friction, the effect of expansion joints in the bridge floor must be taken into account. Furthermore, the relative displacement of stringer and floor beam is greatly affected by the lateral deflection of the floor beams caused by the horizontal friction forces. This deflection obviously reduces or completely offsets the relative displacements of stringers and floor beams. In computing the frictional forces, only those stringers that are located at points of the floor beams which cannot deflect laterally if acted on by friction forces may be considered as

contributing to the energy dissipation. In Fig. 6 the solid lines indicate such stringers, whereas the stringers shown by the dashed lines may cause distortion of the top chord of the floor beams, comparable in magnitude with the relative displacement between stringer and floor beam.

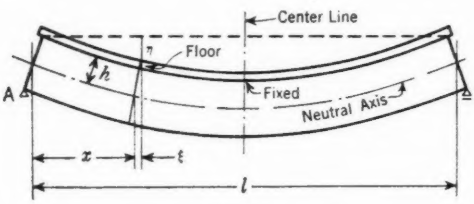


FIG. 7

The procedure for deriving the damping capacity of the bridge resulting from friction between floor and floor beams will be demonstrated under the assumption that the bridges vibrate in the first symmetric mode. The floor first may be considered tied to the trusses at midspan only, as indicated in Fig. 7. The relative displacement ξ at the distance x from the left support is

$$\xi = h \frac{d\eta}{dx} \qquad (22a)$$

in which η is the displacement form of the first symmetric mode; that is, $\eta = a \sin \pi x/l$ when the bridge vibrates with the amplitude a. Therefore,

$$\xi = \frac{\pi h}{l} a \cos \frac{\pi x}{l} \qquad (22b)$$

The graph in Fig. 8 computed from Eq. 22*b* and referred to as the $\bar{\xi}$-diagram shows the relative displacement ξ of stringer and floor beams in any reference point x when the bridge vibrates with the amplitude $a = 1$.

This diagram, however, does not represent the true relative displacements $\bar{\xi}$ because Eq. 22*b* has been derived under the assumption that the stringers move freely relative to the floor beams in each half span. At the expansion joints (designated E in Fig. 8) each stringer section is tied at its right end to the floor beam. Accordingly, the displacement $\bar{\xi}_f$ of the tied right-hand end of the section must be subtracted from $\bar{\xi}$. The reduced displacement, therefore, is $\bar{\xi}' = \bar{\xi} - \bar{\xi}_f$, and is represented by the ordinates of the shaded areas in Fig. 8.

With w_f denoting the weight per unit length of the floor structure and concrete slab pertaining to those stringers (which are assumed to slide on the floor beams) the friction ΔF per unit length is

$$\Delta F = w_f f \qquad (23)$$

Since $\Delta F\, x\, \bar{\xi}'\, a$ is the work done per unit length of bridge by the friction forces ΔF during one quarter cycle of vibration the total energy dissipated during a complete cycle is

$$\Delta W = 4\, \Delta F\, a \int_o^l \bar{\xi}'\, dx \qquad (24)$$

The integral extends over the total length of the bridge; and $\int_o^l \bar{\xi}'\, dx$ is represented by the sum of the shaded areas.

The maximum value of the total strain energy accumulated in the vibrating trusses in the case under consideration is given by Eq. 14, when I is the moment of inertia of all trusses. Finally the damping capacity ψ of the bridge structure is

$$\psi = \frac{\Delta W}{W} = \frac{16\, S\, l^3\, \Delta F}{\pi^4\, E\, I\, a} \quad \text{(25)}$$

in which

$$S = \int_0^l \xi'\, dx \quad \text{(26)}$$

is the area of all shaded parts in Fig. 8.

Eq. 25 indicates that ψ is inversely proportional to a; and the damping caused by friction between stringers and floor beams is of the first type. A similar equation will be found for all other symmetric and asymmetric modes.

It is easily observed from an inspection of Fig. 8 that reduction of the distance between the expansion joints, for example, to one half of the distance shown in Fig. 8, would diminish the magnitude S in Eq. 25 to about one half, and, accordingly, would reduce the logarithmic decrement δ in the same ratio. On the other hand, the damping capacity could be increased considerably if the distance of the expansion seats could be adequately increased.

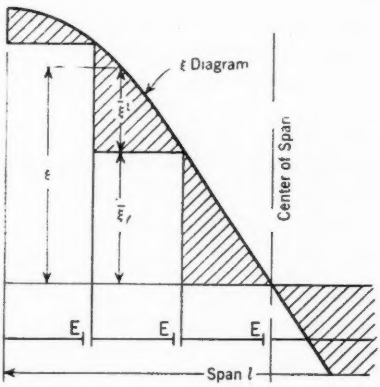


FIG. 8

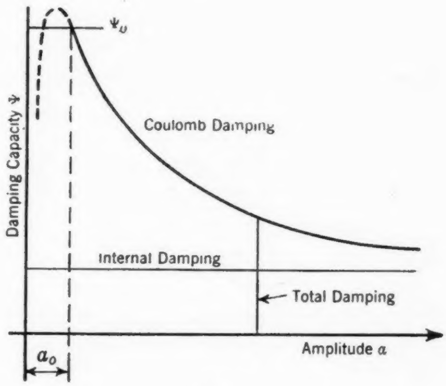


FIG. 9

The purpose of the preceding remark is to direct attention to the fact that, if careful consideration is given to the design of the floor structure in order to utilize a potential source of friction, a substantial amount of additional damping can be provided in suspension bridges without resorting to special structural devices. Most of the foregoing problems show frictional damping of the first type. The total damping capacity ψ—that is, the sum of internal and frictional damping—can be expressed in the form:

$$\psi = C_1 + \frac{C_2}{a} \quad \text{(27)}$$

in which C_1 and C_2 are constants, when small vibrations are assumed. Plotting ψ against the amplitude a leads to the hyperbolic diagram as shown in Fig. 9.

The limiting value ψ_o corresponds to the amplitude a_o which is defined by that amplitude at which static friction is overcome. Since ψ_o defines the maximum amount of the damping capacity of the structure at the beginning of an excited vibration, a brief discussion of the determination of ψ_o in the somewhat involved case of friction in the bridge floor may be in order.

Eq. 25 has been derived under the tacit assumption that the amplitude a had reached a value where relative displacement of stringers and floor beams occurs at all points of the span. Actually, such is not the case at the beginning of a forced or excited oscillation. The longitudinal shear stresses between the floor structure and the floor beams vary along the span, from which it follows that, at small amplitudes, sliding already may have started in a part of the span while in the remaining part static friction may still prevent any movement. To obtain the relationship between the logarithmic decrement δ and the amplitude a during the state of motion under discussion it would be necessary to calculate, step by step, the consecutive values which δ assumes with increasing amplitude a, until the maximum value δ_o is reached, when friction is overcome in all parts of the span.

A rough estimate of the magnitude of δ_o at the instant when damping already has set in along the entire span can be obtained as follows:

Presume that the longitudinal shear stresses in all parts of the span have the same value, assumed equal to the average value of the actual stresses. The amplitude a at which sliding starts simultaneously at all floor beams follows immediately from the simple relation:

$$\Delta F = s\, a_o \qquad (28)$$

s being the constant shear stress per unit length of span which corresponds with the amplitude a equal to unity. The value of a at which damping sets in, therefore, is

$$a_o = \frac{\Delta F}{s} \qquad (29)$$

Since, by supposition, damping is present at all points of the span, Eq. 25 applies and furnishes (upon introducing a_o from Eq. 29, the maximum value ψ_o—

$$\psi_o = \frac{16\, S\, l^3\, s}{\pi^4\, E\, I} \qquad (30)$$

The foregoing step-by-step development of friction in the floor structure suggests a form of the damping diagram between zero amplitude and amplitude a_o as indicated by the dashed part of the curve in Fig. 9.

Corroboration of Theory by Tests.—The careful tests made on a model truss, discussed in Part II, afford a convenient opportunity to compare the prediction of the previously developed theory with the outcome of the experiments. Interpretation of these tests in the light of the theory is represented in Part III.

PART II. DAMPING TESTS ON SIMPLE BEAMS AND TRUSSES

The experimental study of the damping characteristics of structural members reported in this paper was made at the Research Laboratories of the

United States Bureau of Public Roads in Arlington County, Virginia, at the request of the Advisory Board on the Investigation of Suspension Bridges.

The initially authorized program had for its object a determination of the relative damping to be expected in solid members and in trussed structural members, information which has an important bearing on the testing of full models of suspension bridges for aerodynamic action in an air stream. As frequently happens, however, the results obtained in the work initially contemplated led to some expansion of the program so that other useful information on structural damping might be obtained while the experimental facilities were available. The work (first authorized in 1943 and expanded in 1945) has been the subject of three progress reports to the Advisory Board which, at its May, 1948, meeting, authorized the use of the data in the present paper in order to make the information available to the profession.

Scope of the Program

Four main subjects defined for investigation were as follows:

1. The damping characteristics of a trussed structural member in comparison with those of solid members of equal stiffness, when vibrated in the first symmetric mode as a simply supported beam. (The tests were planned to indicate the damping over a considerable range of dynamic stress.)
2. In the case of a bolted truss, the effect of bolt type, bolt tension, and of the procedure used in developing a given degree of bolt tension.
3. The effects of a variation in dead load on the damping characteristics of the bolted truss.
4. The effect of dry or Coulomb friction, externally applied, on the damping characteristics of the bolted truss. (The Coulomb friction was developed in three different ways: By mechanical brakes, by sliding bearings at the ends of panels of a simulated floor system, and by sliding end bearings as a replacement for ball-bearing wheels.)

To control the testing procedures properly, and for other reasons, certain collateral investigations were required. Among these were:

5. The establishment of relations between load, deflection, and strain in the specimens.
6. A determination of the ultimate strength and character of failure of the chord splice in tension, and the chord section in compression, as used in the bolted truss.
7. A determination of the maximum tightening torque for the bolt-nut combinations used in the truss.
8. Segregation of the air damping effect when a simulated floor system was applied to the truss.

Description of the Tests

The Specimens—Three types of specimens were used in the tests: (*a*) A trussed member with bolted joints, (*b*) a rolled **H**-section, and (*c*) a solid bar of rectangular cross section. The truss (as it will be called) was made up of angles 1 in. by 1 in. by 0.065 in. in section, cold-formed from a low alloy steel of relatively high strength. The **H**-section and the rectangular bar were hot-

rolled, of low carbon steel. The three specimens were designed to have equal moments of inertia, although, as received, the moment of inertia of the truss was slightly greater and that of the H-section, somewhat smaller than the moment of inertia of the rectangular bar. The principal dimensions of the cross section of each specimen as received are shown in Fig. 10.

For testing, each specimen was placed horizontally as a simply supported beam having a rocker bearing at one end and a wheel bearing, rolling on balls, at the opposite end. The truss and the H-section were compared on a span of 36.83 ft. The truss and the rectangular bar were compared on a span of 28.33 ft, the truss being shortened by the removal of a section, for these tests.

Certain pertinent information as to the weight, stiffness, and other physical characteristics of each specimen are given in Table 1.

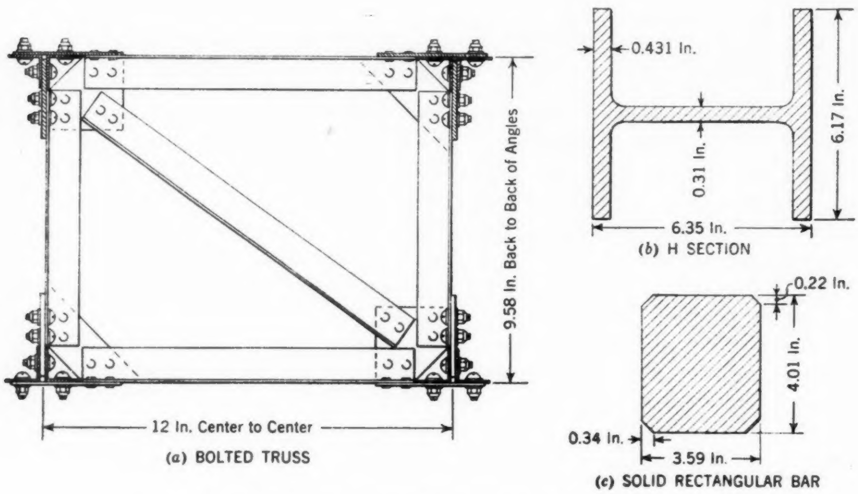


Fig. 10.—Cross Section Dimensions of Specimens

In order that the weight of the truss would equal that of the H-section or that of the rectangular bar as required, dead weight was added to the truss at the lower panel points. The dead-load weights bolted to the truss at the lower panel points may be seen in Fig. 11 which shows also the general appearance of the shortened truss as mounted on the 28.33-ft span.

Test Methods and Instrumentation.—The general test procedure has been to excite the specimen (mounted as a simple beam) to a given degree of oscillation in the first symmetric mode in the vertical plane and, having developed oscillations of the desired amplitude, to allow these to die out naturally through damping caused by (1) internal friction, (2) end bearing friction, and (3) air friction. Since end bearing friction may be assumed to be constant for specimens of equal weight and since air friction could at best exert only a very small damping force under the conditions of these tests, it was believed that any important differences found in the damping of the different specimens would be the result of differences in the damping forces within the specimens.

The rate of energy absorption or damping can be determined from the amplitude decay through the use of a quantity called the logarithmic decrement δ, introduced in Part I. This quantity may be defined as the natural logarithm of the ratio of the amplitudes of any two successive oscillations. For convenience, in these tests single amplitudes (that is, movements to one side of the mean position) have been used for decrement determination.

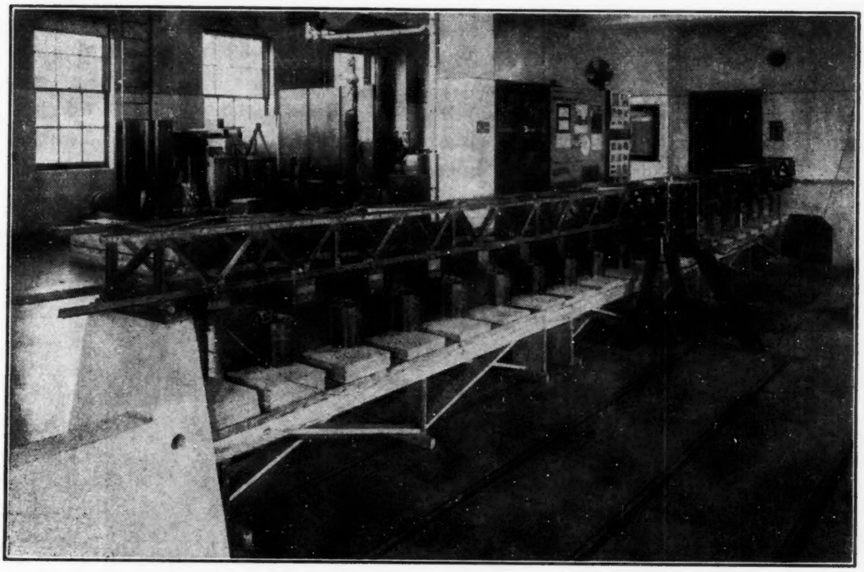


FIG. 11.—BOLTED TRUSS; SPAN, 28.33 FT AND WEIGHTS 51.75 LB EACH

TABLE 1.—COMPARISON OF PHYSICAL PROPERTIES OF SPECIMENS AS TESTED

Description	SPAN LENGTH 36.83 FT		SPAN LENGTH 28.33 FT	
	Truss	H-section	Truss	Rectangular bar
Moment of Inertia (Inches^4):				
From cross section	20.3	16.9	20.3	19.0
From load deflection	20.7	16.6	20.4	18.9
Weight per Foot (Pounds):				
Specimen proper	7.26	24.00	7.59	48.26
Attached dead-load weight	21.13	0.16	41.96	0.36
Total dead weight	28.39	24.16	49.55	48.62
Maximum deflection from dead weight (inches)	2.05[a]	2.05	1.24[b]	1.27
Natural period (second)	0.39	0.41	0.31	0.32

[a] Permanent set of 1.09 in. developed during testing. Total deflection at end of test, 3.14 in. [b] Permanent set of 0.71 in. developed during testing. Total deflection at end of test, 1.95 in.

The specimen under test was made to oscillate in the vertical plane by the simultaneous release of a number of symmetrically arranged cylindrical sus-

pended weights of 19.5 lb, 34.5 lb, or 51.75 lb each, depending on the test condition for which the specimen was used. To distinguish them from dead-load weights that were attached to the specimens for various purposes, these suspended weights have been termed "live-load" weights. They were supported by steel hooks extending downward from the specimen at 17-in. intervals, a distance determined by the panel length of the truss. On the 36.83-ft span, twenty-six live-load weights were used, whereas on the 28.33-ft span the number was twenty.

By means of a solenoid-actuated trigger system, all the live-load weights were released simultaneously by closing one switch, making it possible to duplicate the excitation of the specimen for as many times as the program required.

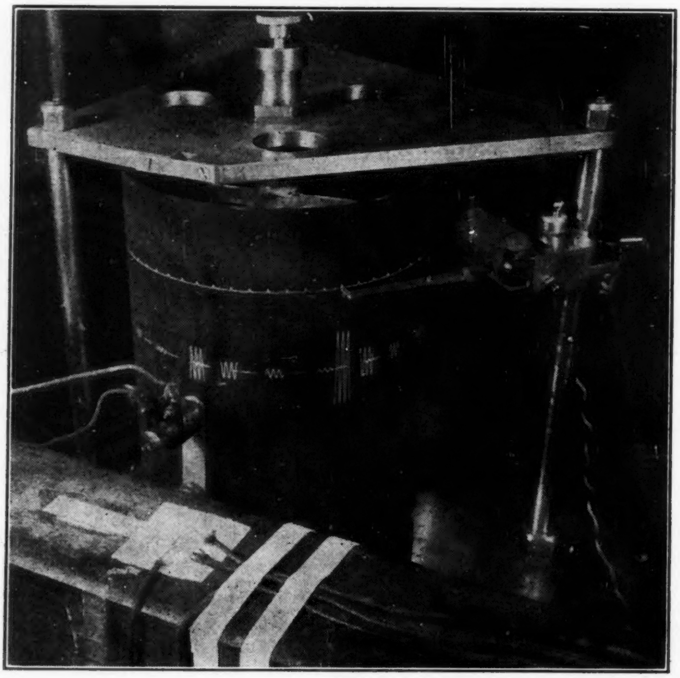


FIG. 12.—AMPLITUDE RECORD, SOLENOID-ACTUATED STYLUS AND STRAIN GAGE INSTALLATION-BAR

In most of the tests the amplitude of the oscillation was recorded at midspan by a sharply pointed steel stylus attached to the specimen in a position such that it could be brought into light contact with a smoked paper wrapped around a slowly revolving drum. The stylus, under automatic control, periodically traced on the smoked paper complete cycles of the vertical motion of the oscillating specimen. Usually these traces were made at the beginning of the oscillation and during the first second of each 10-sec interval thereafter. A time trace with 1-sec intervals was placed on the record by another electrically-actuated stylus controlled by a calibrated pendulum. The details of this deflection recording equipment are shown in Fig. 12.

In some of the later tests with the truss where the damping rate was high, the intermittent recording of amplitude decay could not be used and a different method for recording was adopted. The method employed a magnetic displacement pickup with a galvanometer type oscillograph for recording, which made possible continuous recordings of oscillation decay with no physical contact between the test specimen and the recording medium.

Electrical resistance type strain gages were placed on each specimen at midspan to provide information regarding maximum stresses for the various conditions of test. One of the gages on the upper surface of the rectangular bar may be seen in Fig. 12. The relations between strain and load and between deflection and load were used for a number of purposes during the course of the program.

Comparison of the Internal Damping Characteristics of Trussed and Solid Steel Members.—As originally fabricated the joints in the truss, except for the lateral bracing in the plane of the upper chords, were drawn up with No. 10–24 commercial machine screws used as bolts. These passed through clearance holes in the gusset plates and truss member and were fitted with washers and hexagonal nuts of commercial grade. This was the type of joint fastening used in the initial tests.

By a number of tests with a torque-indicating wrench, it was found that this screw and nut combination would withstand a tightening torque of 40 in.-lb or more before failure occurred. On the basis of this information it was decided to test the truss at three degrees of bolt tension, measured by tightening torques of 35 in.-lb, 25 in.-lb, and 15 in.-lb, respectively. The net section of a No. 10–24 machine screw is approximately one half that of a rivet of the same over-all diameter.

The truss bolted up with the No. 10–24 machine screws tightened with a torque of 35 in.-lb was compared with the H-section on the 36.83-ft span, the excitation being from the release of the 19.5-lb and the 34.5-lb live-load weights. The shortened truss was compared with the rectangular bar on the 28.33-ft span, the excitation being from the release of 19.5-lb, 34.5-lb, and 51.75-lb live-load weights. Under the various test conditions initial double amplitudes at midspan varied from about 0.7 in. to about 4.1 in. and the maximum stress in the extreme fiber at midspan ranged from about 6,350 lb per sq in. to about 23,450 lb per sq in.

The measurement of both strain and deflection at midspan made it possible to relate midspan deflection to extreme fiber stress for each specimen and from this to relate instantaneous logarithmic decrement values to the maximum fiber stress developed during the particular oscillations to which they apply. Graphs showing this variation of logarithmic decrement with dynamic stress were particularly useful for the comparisons desired in this investigation.

Fig. 13(*a*) shows decrement-stress relations for the truss and for the H-section tested under identical conditions on the 36.83-ft span. The initial amplitudes cause the highest stresses and appear toward the right-hand side of the graph. When excited with the 34.5-lb live-load weights, the truss showed an initial decrement value of 0.0516 as compared with a value for the H-section of 0.0070. As the amplitudes decrease the dynamic stresses and the decrement

values also decrease to a value of the general order of 0.0042 for the truss and 0.0036 for the solid section.

Fig. 13(*b*) shows a similar comparison between the shortened truss and the rectangular bar tested under identical conditions on the 28.33-ft span, three different live-load weight values being used to excite the oscillations. Even

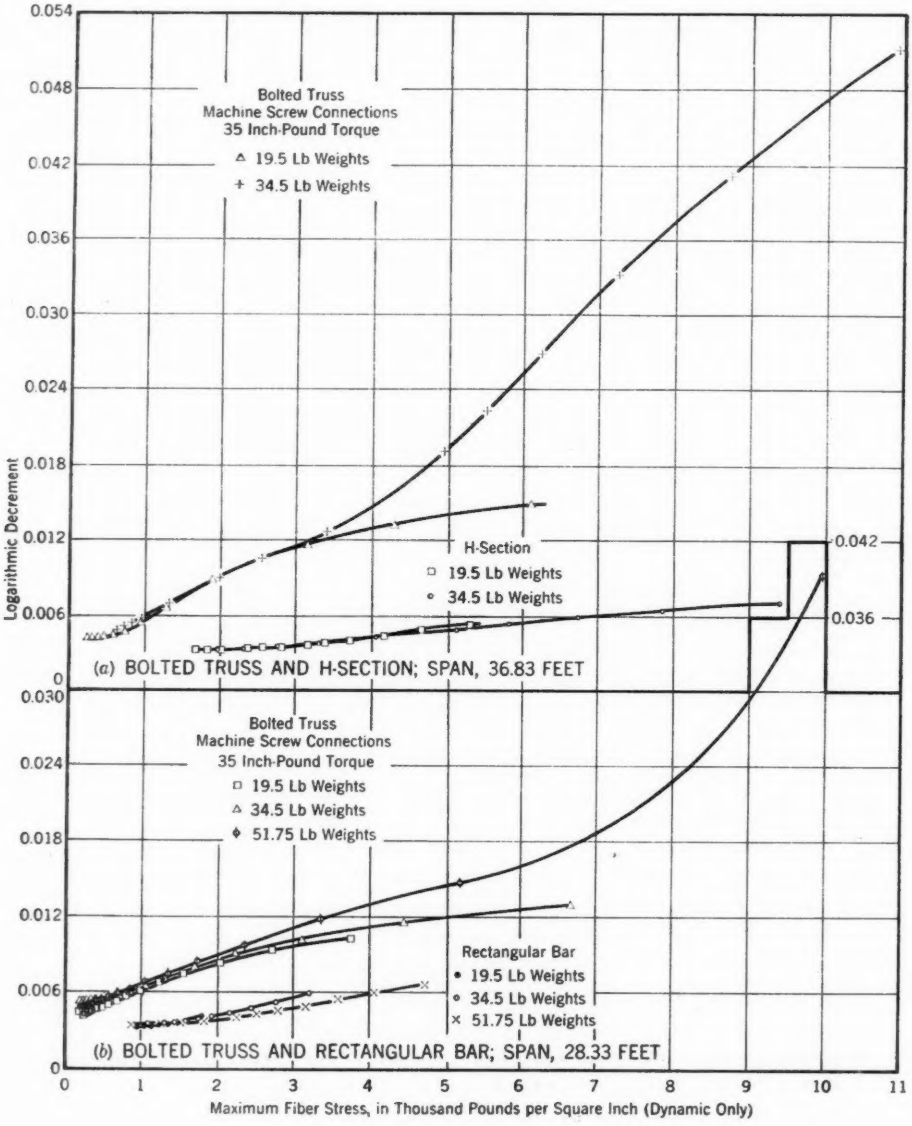
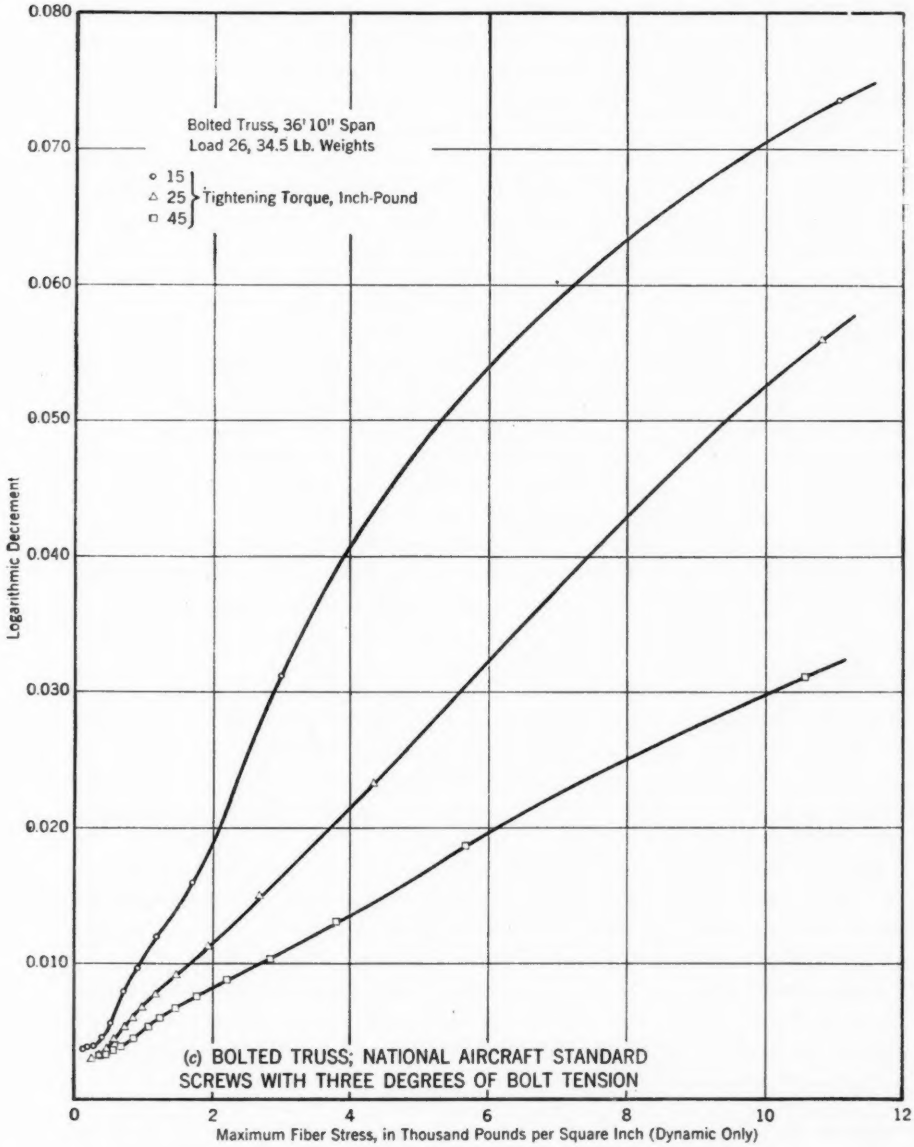


FIG. 13.—VARIATION OF LOGARITHMIC

with the 51.75-lb weights the maximum dynamic stress in the bar was only about 4,700 lb per sq in. and at this stress level the decrement value for the truss was approximately twice that for the bar. In this comparison as in the previous one the maximum damping of the truss is found near the beginning of the test where the stresses and amplitudes are greatest. It is under these conditions



Decrement δ with Fiber Stress

that the likelihood of motion in the joints, with accompanying dry friction, is greatest.

Each point on these and other graphs of the same type represents an average of five to seven tests and may be considered quite well established since the variation for a given set of test conditions was generally quite small.

In these initial tests (which involved the truss fastened with the No. 10–24 machine screws) a study was made of the damping of the truss as influenced by bolt tension. As might be expected, it was found that the more tightly the joint is bolted up the less will be the damping, particularly at the higher stresses. Since the net section of the No. 10–24 machine screw was about half that of the hole through which it passes, the question arose as to what might be the damping characteristics of the trussed specimen if the bolts more nearly filled the holes. Examination of some of the joints showed that there had been relative motion of the bolted members and that in some cases the threads along the body of the bolts had been considerably deformed by it.

Further tests were made with bolts that had a solid body where the bolt passed through the plates and which fitted closely in the hole through these plates. These bolts were of high strength steel manufactured to close tolerances, with well formed threads (32 per in.) and were available with solid (unthreaded) bodies of various lengths. They are designated National Aircraft Standard (NAS) screws. The nuts used with the NAS screws were a commercial product known as the "elastic stop nut," each nut being provided with an integral fiber washer which tends to prevent the nut from working loose. Tests with a torque wrench showed that, as used in these tests, the torque required to turn the nuts (in the new condition) on the threads of the bolt was of the order of 1 in-lb to 2 in-lb. Tests of the NAS screw-elastic stop nut combination showed a failure of the bolt in tension at a tightening torque of about 90 in.-lb as compared with about 45 in.-lb for the commercial machine screws used in the initial tests.

For the tests of the truss with the NAS screws, tightening torque values of 15 in.-lb, 25 in.-lb, and 45 in.-lb were selected. The first two gave a direct comparison with the data from tests with the machine screws, whereas the 45 in.-lb value was selected because tests indicated that it caused a tensile stress of about 60,000 lb per sq in. in the bolt body, a value which seemed to be a reasonable upper limit for structural bolting.

Having reamed the 1,700 holes in the bolted connections of the truss and replaced the machine screws with the NAS screws, tests were made with the truss on the 36.83-ft span with the 34.5-lb live-load weights, using tightening torques of 15 lb, 25 lb, and 45 lb. Typical data are shown in Fig. 13(*c*).

From these comparisons of bolt type and bolt tightness it would appear that in so far as self-damping characteristics of the truss are concerned the tightness with which the connections were bolted is of more importance than is the fit of the bolts in the bolt holes.

Effects of Variation in Dead Load on the Damping Characteristics of the Truss.—One of the questions raised in the discussion of the damping tests by

the Advisory Board had to do with the possible effects of changes in dead load on the damping characteristics of the bolted truss. It was particularly desired to learn what might be the effect of an increase in dead load. To obtain a relatively wide spread in dead-load values without developing an excessive combined live and dead-load stress in the chord members the dead load of the truss was first reduced from its normal value of 1,059.6 lb to 843.5 lb by decreasing the dead-load weights attached at the lower chord panel points. After tests were made with this dead-load value, using both the 19.5-lb and 34.5-lb live-load weights, weights were added to the truss to bring the dead load up to 1,278.2 lb, and the tests were repeated.

Amplitude decay data for each of the four test conditions are shown in Fig. 14. The tests indicate that increasing the dead load of the truss by 51.5% caused no significant change in its damping characteristics.

Damping of the Truss Increased by Dry Friction Externally Applied.—The behavior of the truss in the early tests led to a discussion of the probable behavior of a truss equipped with a deck having sliding bearings to support the deck sections. As a result the program was extended to include some study of the effects of augmenting the dry friction within the truss with dry friction

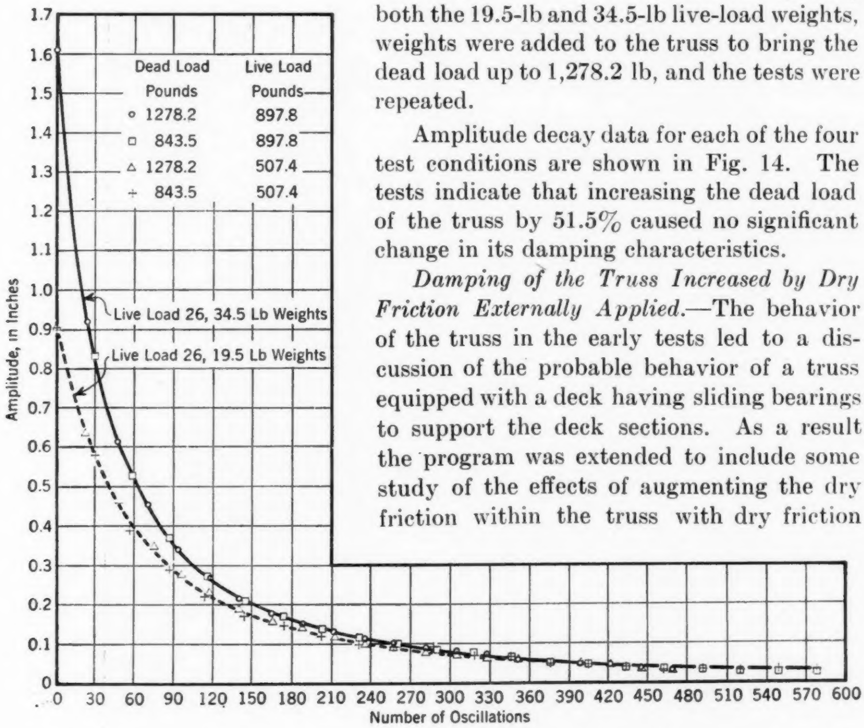


Fig. 14.—Effect of Dead-Load Variations on Amplitude Decay; Bolted Truss: Span, 36.83 Ft, and National Aircraft Standard (NAS) Screws with a Tightening Torque of 45 In.-Lb

developed without, by (*a*) mechanical brakes, (*b*) sliding bearings at the ends of sections of a simulated floor system, and (*c*) sliding end bearings at the truss support.

A mechanical brake was developed which made it possible to apply dry or Coulomb friction in controlled amounts at each end of the oscillating truss. The design for the brake is quite simple and may be readily understood by reference to Fig. 15.

As the truss oscillated in the vertical plane the end frame went through a rotational motion causing a horizontal sliding motion at the contact between the friction surfaces. With the three weights used on the platform of the brake, vertical reactions of 70 lb, 140 lb, and 210 lb were developed on the braking

surfaces. One of these brakes was installed at each end of the truss and both were operative in all tests.

Amplitude decay curves obtained with the mechanical brakes in operation are shown in Fig. 16(*a*) in comparison with that for the truss with no external dry friction. These data were obtained with the bronze lower friction surface in contact with cold-rolled steel upper friction surfaces of the three types used in the tests, and for each three pressure intensities.

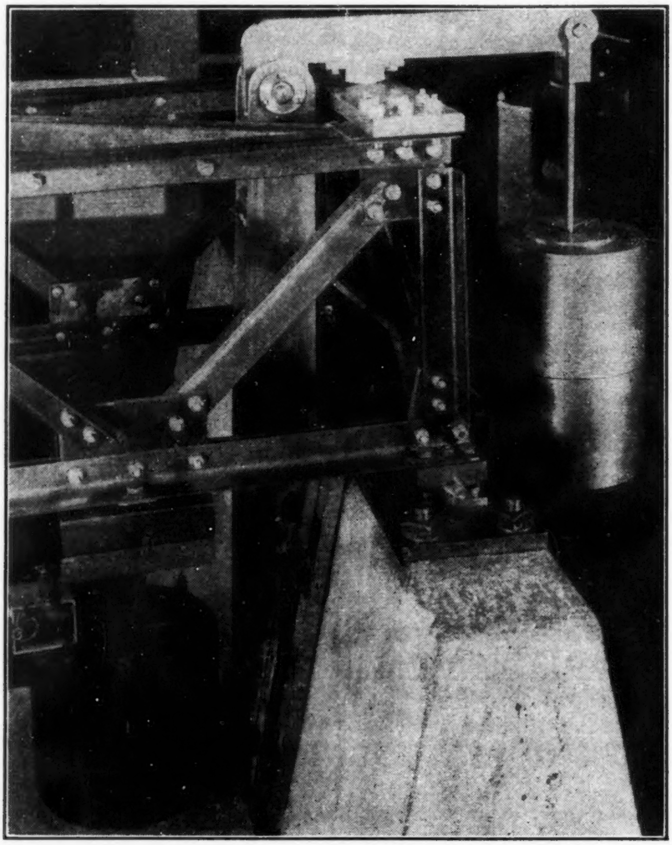


FIG. 15.—FRICTION BRAKE, SHOWING THE CYLINDRICAL SURFACE ON THE BRAKE LEVER IN CONTACT WITH THE PLANE SURFACE ON THE TRUSS

It is apparent from Fig. 16(*a*) that, where bronze and steel plates were used as opposing friction surfaces, the braking action was smooth and consistent, the damping increased progressively with the force (F_n) applied normally to the surfaces, and the changes in the area of the friction surfaces had little effect. In other words the laws of dry friction, or Coulomb friction, controlled.

In contrast, it was found that, where the opposing friction surfaces were of steel, even though one was much harder than the other, abrasion and scoring

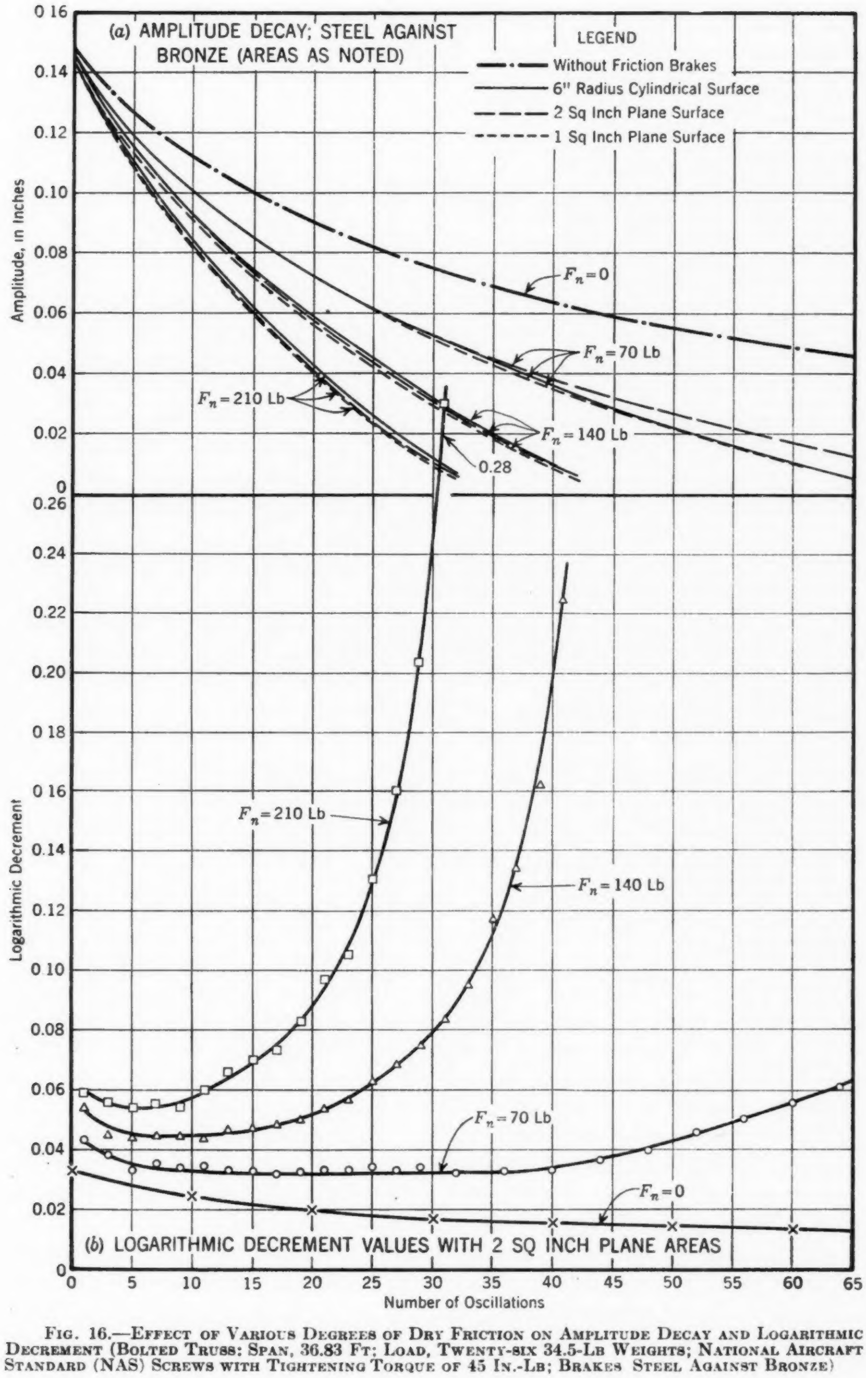


FIG. 16.—EFFECT OF VARIOUS DEGREES OF DRY FRICTION ON AMPLITUDE DECAY AND LOGARITHMIC DECREMENT (BOLTED TRUSS: SPAN, 36.83 FT; LOAD, TWENTY-SIX 34.5-LB WEIGHTS; NATIONAL AIRCRAFT STANDARD (NAS) SCREWS WITH TIGHTENING TORQUE OF 45 IN.-LB; BRAKES STEEL AGAINST BRONZE)

were prevalent. The result is that, although the damping action at times was greater than with the bronze-steel combinations, the action was neither consistent nor dependable, and quite unsuited for test purposes.

It is interesting to note that the behavior of these friction surfaces is similar to the action observed in some bridge bearing-plate friction tests[3] reported by the Bureau of Public Roads in 1936. It was concluded from these earlier tests that combinations of like or unlike ferrous metals gave the highest, and that ferrous metals in contact with bronzes gave the lowest, coefficients of friction. Also, where the opposing plates were of ferrous metals, either like or unlike, seizure and scoring almost always occurred.

Thus the evidence indicates that where sliding plates are to be used in an unlubricated condition to develop friction for damping, more dependable action will be obtained if the opposing plates are of bronze and steel rather than of two ferrous metals.

It is evident from an examination of Fig. 16(*a*) that, with the mechanical brakes acting, the amplitudes decrease very rapidly to values of the order of 0.01 in. or less, as measured at the deflection gage position near the end of the truss, and that the shape of the amplitude decay curve tends to become linear as the braking force is increased. This is as would be expected since the Coulomb damping from the braking system tends to become the dominant component as this force is increased.

Fig. 16(*b*) shows logarithmic decrement values computed from the amplitude decay curves of Fig. 16(*a*) for the 2-sq in. friction plate area, and for four values of the braking force as measured by the normal component F_n. The complex nature of the over-all damping with the brakes acting made it expedient to compute decrement values directly from successive measured amplitudes at frequent intervals. This accounts for the slight scatter of the points for the three upper curves of Fig. 16(*b*).

Following the completion of the tests with the friction brakes at the ends of the truss, tests were made in which external dry friction was developed in a simulated deck system.

This deck was designed as a series of steel plates $7\frac{3}{4}$ in. wide by $\frac{3}{16}$ in. thick arranged end to end the full length of the truss. Except for one section 34 in. long at midspan the deck sections were each 8.5 ft long. At alternate panel points of the upper chord small plates of cold-rolled steel were bolted transversely on the truss and on these the deck sections were supported. At one end of each individual deck section (except the 34-in. section) two short vertical legs were fastened. This pair of legs was attached to one of the transverse plates by knuckle bolts, making a connection that was resistant to horizontal or vertical movement, but not to rotation. At each of the other support points, the deck section was fitted with a similar pair of short legs except that these were tipped with bronze shoes at the lower end to provide horizontal sliding friction bearings as the truss oscillated in the vertical plane.

To develop the desired deck load of approximately 500 lb, and at the same time maintain the total dead weight in the specimen, the dead-load weight

[3] "Determination of Coefficients of Friction of Sliding Bearings for Bridges," by George W. Davis, *Public Roads*, December, 1936, p. 223.

attached at the lower chord panel points was reduced from 768.9 lb to 274.3 lb, a difference of 494.6 lb. Ballast weights attached to the deck sections at the support points were used to bring the weight of simulated deck up to an amount

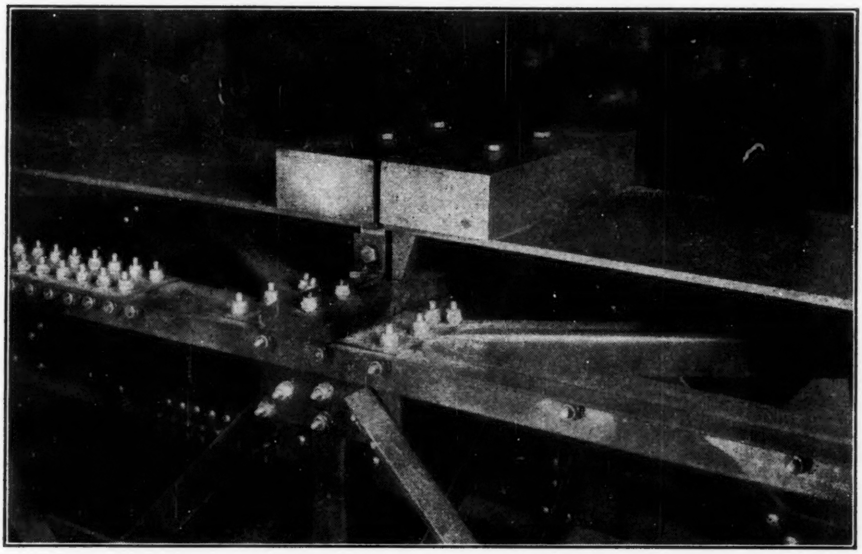


FIG. 17.—FIXED AND SLIDING BEARINGS AT THE ENDS OF STEEL PLATES OF SIMULATED DECK

approximately equal to that removed from the lower chord panel points. The steel deck, supports, and ballast weighed 497.7 lb, making the total dead-load weight 1,062.6 lb as compared with 1,059.6 lb before the deck was added.

In Fig. 17 are shown the details of the fixed-end connection of one deck section, the sliding bearing of the adjoining section, and the manner in which the ballast weight was applied to the plates of the deck over the support points. It has been recognized from the beginning that there has been a certain amount of air damping present in the over-all damping for which logarithmic decrement values have been determined. In view of the generally low decrement values it has been thought that the air damping probably was not an important element in the tests of the truss without the deck. With the addition of a relatively large horizontal area such as was represented by the simulated deck there was reason to suspect that the amount of air damping might become an important component of the over-all damping and some experiments were made to determine how important this element might be.

A series of deck plates of $\frac{1}{4}$-in. plywood were attached to the truss in its original form. Each plywood plate was attached to the truss only at the mid-length of the plate and as the plates were separated slightly, no stiffening effect was introduced. The total area of the plywood sections was the same as that of the steel deck. The total weight of the plywood was 16.75 lb.

Fig. 18 shows the relation between the logarithmic decrement and the maximum dynamic fiber stress in the chord members for the three deck conditions (no deck, plywood plates, and the steel deck with friction shoes acting). This graph is of the same form as Fig. 13, with the high stress values measured at the beginning of the test shown at the right-hand side of the plot. The decrement values were computed from measured values of successive amplitudes in the manner described in connection with the friction brake tests. This probably accounts for the slight scatter of the points on the curve for the steel deck condition in Fig. 18.

It is apparent that the additional air resistance created by the application of the plywood plates, and the consequent addition of some 24 sq ft to the frontal area, does increase the over-all damping to a degree that is measurable, although the effect is relatively small. This tends to support the assumption that the air resistance of the truss alone was not an important component of the over-all damping measured for that condition.

With the steel deck sections added, and with the Coulomb friction at the sliding supports in operation, the increase in total damping was found to be relatively large, however, being quite similar to the effect observed with the brakes acting at the ends of the truss and corresponding approximately to the damping observed with a braking force $F_n = 70$ lb.

As might be expected, the frequency of the truss oscillations changed slightly after the steel deck sections were installed, the effect being to raise the apparent

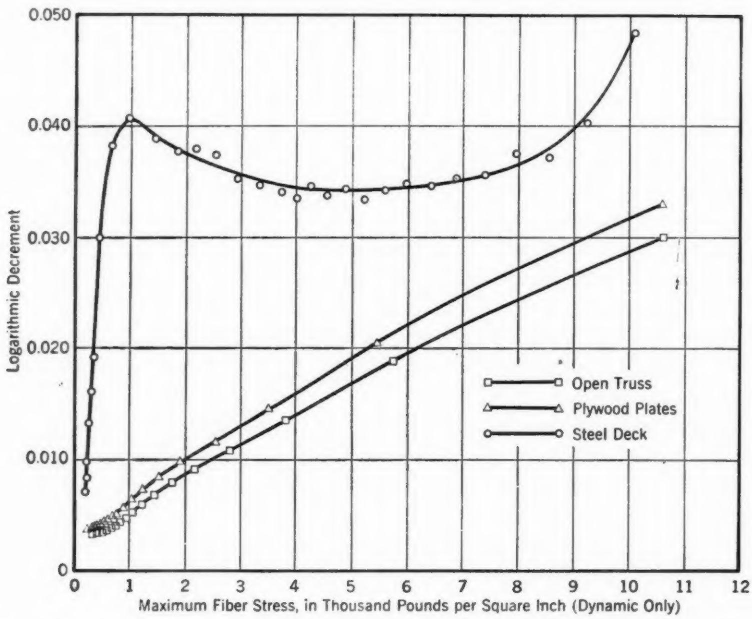


FIG. 18.—VARIATION OF THE LOGARITHMIC DECREMENT δ WITH FIBER STRESS FOR THREE DECK CONDITIONS (BOLTED TRUSS: SPAN, 36.83 FT; LOAD, TWENTY-SIX 34.5-LB WEIGHTS; NATIONAL AIRCRAFT STANDARD (NAS) SCREWS WITH A TIGHTENING TORQUE OF 45 IN.-LB)

moment of inertia about 3%. Also, it was found that the change in frequency was not constant but increased slightly as the amplitudes became smaller.

The third method used for developing external Coulomb friction included in these tests was to substitute sliding shoes for the ball-bearing wheels that supported one end of the truss in most of the tests. The shoes were mounted on ball bearings carried by the same cross shaft that normally was used for the wheel bearings so that no additional rotational resistance was added to the system by the change. The lower surface of each shoe was a smooth bronze plate 3 in. long by 0.6 in. wide and the opposing surface was of cold-rolled steel.

Comparative amplitude decay data are shown in Fig. 19, which are the curves for the truss on the wheel end bearings, for the truss on the sliding end bearings, and for the truss on the wheel bearings with the friction brakes in operation under a normal force, F_n of 210 lb per brake. It is apparent that the sliding end bearings developed a very strong damping force as the truss oscillated—stronger even than that with the two friction brakes operating. It will be recalled that the truss weighed about 1,050 lb so that the force exerted vertically on the end bearings was about 525 lb as compared with a total vertical force of 420 lb on the surfaces of the brakes.

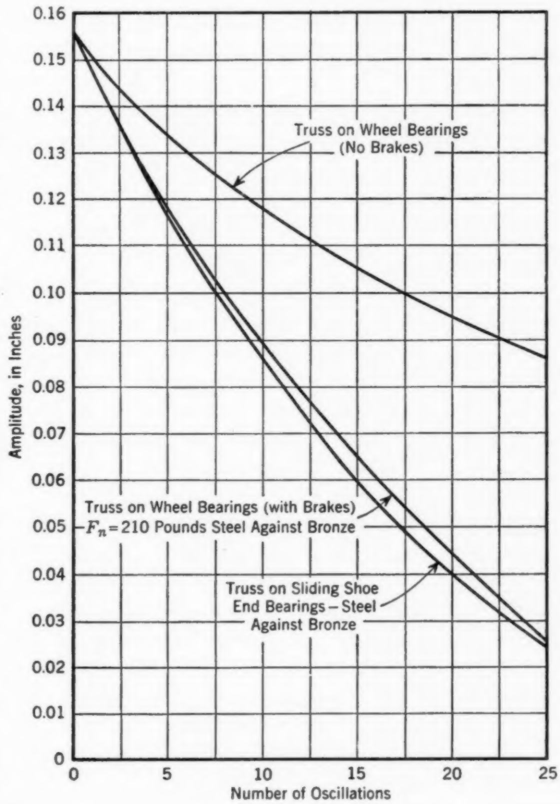


Fig. 19.—Comparative Effect of Brake Friction and Sliding End Bearing Friction on Amplitude Decay

For a better understanding of the forces that were being developed in the tests, in which external dry friction was being applied to the truss, it seemed desirable to obtain data on the frictional action of the bronze on steel surface combination, in the unlubricated condition, when the moving part was sliding with a horizontal reciprocating motion.

Accordingly, an apparatus was designed and constructed in which the force normal to the friction surfaces was developed with the same lever arrangement used in the mechanical brake on the truss; and the amplitude and frequency of

horizontal motion of the friction surfaces were approximately the same as obtained when these same surfaces were used in the brakes during the testing of the truss. By means of a strain gage and oscillograph, the entire cycle of the force required to overcome resistance to sliding at the plane of the bronze-steel friction surfaces was recorded.

From tests with this device, using the same bronze and cold-rolled steel surfaces that were used in the friction brakes, it was found that:

a. The horizontal force necessary to cause the friction surfaces to slide with the reciprocating motion as described, varied with time, essentially in the form of a "square" wave;

b. The average coefficient of sliding friction for the plane surfaces of cold-rolled steel on bronze, in the unlubricated condition, as determined by six tests, was 0.141; and

c. Once sliding began, the resistance to motion (or the coefficient of friction) did not vary with the velocity of the sliding motion.

Summary of Indications from the Damping Tests

The analysis of the data obtained from the structural damping tests, made in the manner described, indicates that:

1. The character of the amplitude decay for the two solid sections tests was definitely exponential whereas that for the bolted truss was more complex, being apparently the result of a combination of the imperfect elasticity of the material with plastic yielding and friction due to small displacements in the joints. In these tests, of course, there was end bearing friction and air friction, but, as stated previously, the former was present as a constant and the latter was believed to be relatively unimportant.

2. In comparisons between the solid members and the bolted truss, the truss showed considerably greater capacity for self damping, particularly at the higher amplitudes (or higher dynamic stresses). With the joints tightly bolted, the decrement values ranged from about 0.006 at 1,000 lb per sq in. to about 0.030 at 10,000 lb per sq in. in comparison with a range of 0.0036 to 0.0070 for the H-section at the same limits of dynamic stress.

3. The decrement values for all three specimens increased with an increase in amplitude (or dynamic stress), the rate of increase being greatest with the truss and least with the H-section.

4. For small amplitudes and small dynamic stresses the decrement values were of the general order of 0.004 for all specimens. The decrement values for the solid members did not exceed 0.007 in any of the tests. For the truss, however, values as high as 0.073 were found at high initial amplitudes (and high dynamic stresses) in combination with low bolt tension in the bolted connections.

5. The tightness of the bolted connections had a pronounced effect on the damping of the truss, particularly at the larger amplitudes (or dynamic stresses). The tighter the bolts, the lower were the decrement values.

6. The damping action of the joints in the bolted truss, apparently, was influenced more by the pressure of the surfaces held in contact by the bolts than it was by the fit of the bolts in the holes.

7. A variation in the dead load of the truss of the order of 50% caused no measurable change in its damping characteristics.

8. The external application of dry or Coulomb friction greatly increased the damping of the bolted truss.

9. Where mechanical brakes that employed metal plates rubbing together without lubrication were used to develop frictional forces for damping purposes, more dependable action was obtained with opposing surfaces of bronze and steel than with two ferrous metals in contact.

10. The damping obtained from the rubbing of steel on bronze without lubrication follows the laws of Coulomb friction; that is, it varies directly with the total normal pressure on the friction surfaces, it is independent of their area, and it is nearly independent of velocity. The effective coefficient for the condition of this test was about 0.14.

11. The application of the plywood plates to the truss, in lieu of the steel deck panels of equal frontal area, increased the damping to a degree that could be measured, although the effect of the added air resistance on damping was relatively small.

12. The increase in the damping of the bolted truss, obtained by the application of the simulated deck sections, was quite similar in character to that obtained with mechanical brakes in operation at the ends of the truss.

13. The substitution of sliding end bearings of bronze on cold-rolled steel for the wheel bearings used at one end of the truss developed a strong damping force.

PART III. CORROBORATION OF THE THEORY OF FRICTIONAL DAMPING BY THE EXPERIMENTS

Damping Effect of Mechanical Brakes at the Ends of the Truss

The vibrating system, damped by the friction forces F which act at top chord level at both ends of the truss is shown in Fig. 20. Applying the same reasoning as in the case of damping due to friction in the bearings at one end of a single span, discussed in Part I, leads to the derivation of Eq. 14 with h denoting the distance from the sliding surfaces of the brakes to the neutral axis of the model truss. The friction coefficient f was found, by special tests, to be $f = 0.141$ from which the braking force F (for $F_n = 210$ lb normal pressure) is $F = 210 \times 0.141 = 29.6$ lb.

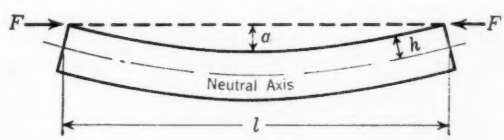


Fig. 20

Introducing this value of F into Eq. 14, with $E = 29{,}600{,}000$ lb per sq in.; $l = 442$ in.; $h = 5.35$ in.; and $I = 20.3$ in.^{4}, gives

$$\psi = \frac{0.0542}{a} \qquad (31a)$$

The logarithmic decrement, therefore, is

$$\delta = \frac{0.0271}{a} \qquad (31b)$$

The theoretical damping hyperbola is shown in Fig. 21, in which δ is plotted

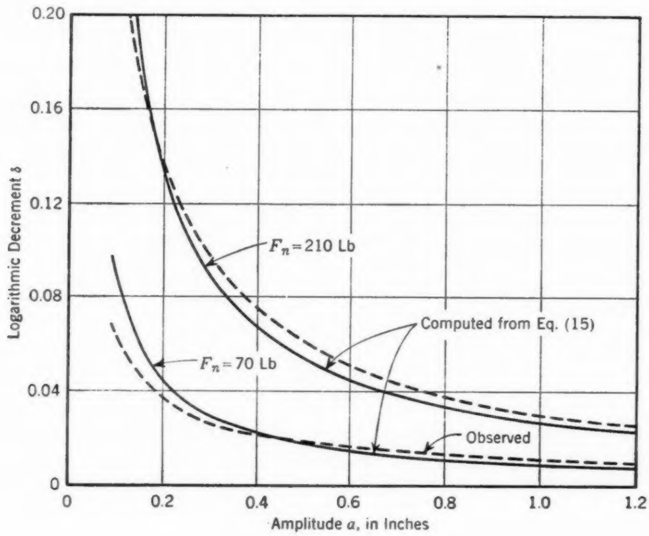


Fig. 21

against the amplitude a. A similar graph is given for $F_n = 70$ lb which follows the equation:

$$\delta = \frac{0.0090}{a} \qquad (31c)$$

From Fig. 16 the two dashed curves in Fig. 21 were developed, showing also the observed values of δ as a function of the amplitude a. These curves represent the effect of pure frictional damping, the effect of internal damping having been eliminated with the aid of the curves for $F_n = 0$ in Fig. 16.

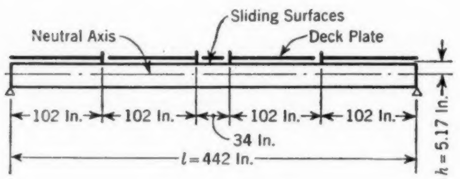


Fig. 22

Fig. 21 indicates good qualitative agreement between theory and experiment. The empirical curves deviate but slightly from a hyperbola. For values of a between 0.4 in. and 1.2 in. the observed values of δ are somewhat greater than the theoretical values, indicating that at least in that range the frictional resistance presumably was 10% to 15% greater than $f = 0.141$ which value underlies the foregoing computation.

Effect of Friction in the Simulated Deck System

A detailed description of the articulated deck system was given in Part II. Fig. 22 shows the arrangement of the deck plate and the location of the expansion joints. The total weight of the deck structure was 498 lb, or 498/442 = 1.13 lb per in. It follows, therefore, that the friction force $\Delta F = 0.141 \times 1.13 = 0.16$ lb per in. Following the procedure as outlined in Part I, the $\bar{\xi}$-diagram was computed by Eq. 22*b* as $\bar{\xi} = 0.0368 \cos \pi\, x/l$ and shown in Fig. 23(*a*). The shaded area S, for the entire truss, is 3.88 in.; and, ψ, according to Eq. 25, is $\frac{16\, S\, l^3\, \Delta F}{\pi^4\, E\, I\, a} = \frac{16 \times 3.88 \times 442^3 \times 0.16}{97.4 \times 29.6 \times 10^6 \times 20.3} \frac{1}{a} = \frac{0.0147}{a}$.

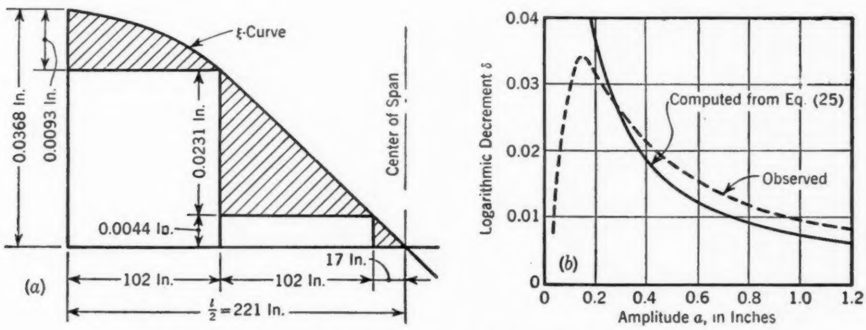


Fig. 23.—Effect of Friction in the Simulated Deck System

The curve $\delta = \frac{0.00735}{a}$ and, for comparison, the observed relationship between δ and a, are shown in Fig. 23(*b*). The latter curve was derived from observed diagrams after eliminating the effect of internal damping. The damping effect as observed, between $a = 0.4$ and 1.2, is distinctly greater than that predicted by the simple formula, Eq. 25. Its shape, showing a peak at $a = 0.15$ in., indicates that the considerations in Part I relating to the beginning of frictional damping in the case of an excited motion (see Fig. 9) are correct in principle.

Acknowledgment

The collapse of the Tacoma Narrows Bridge on November 7, 1940, created a widespread demand for a comprehensive investigation of the dynamic oscillations of suspension bridges. As a result, in 1942, the United States Public Roads Administration sponsored the organization of a group of competent engineers, representing various interested organizations, to explore the problem. This group, whose function it is to assume general direction and supervision of the investigational work undertaken, is known as the Advisory Board on the Investigation of Suspension Bridges. Until his death on February 17, 1950, the senior writer Friedrich Bleich served as consultant to this group. Subsequent to his death, the work of interpreting the original notes, reconciling editorial details, and answering questions raised in discussion concerning Part I of this paper, has been kindly assumed by his son, Hans H. Bleich, M. ASCE.